Generalized Reaction and Unrestricted Variational Formulation of Cavity Resonators—Part II: Nonorthogonal and Free-Boundary Mode-Matching Method

Ali M. Shams-Zadeh-Amiri, *Member, IEEE*, Safieddin Safavi-Naeini, *Member, IEEE*, Sujeet K. Chaudhuri, *Senior Member, IEEE*, and Ramin Sabry, *Member, IEEE*

***Abstract*—This paper addresses a systematic method whereby the conventional mode-matching method is generalized to the cases where the set of modes used for the field expansion within a cavity resonator are relaxed to be orthogonal or satisfy any specific boundary conditions. It is shown that this approach is based on the unrestricted variational formulation of a cavity resonator. Reciprocity theorem and generalized reaction are the mathematical foundations of this new formulation. We have shown that the conventional mode-matching method is a special case of this generalized formulation and indeed is variational in nature. More precisely, we have proven that, if the field distribution obtained based on the conventional mode-matching method is used as a trial one in some variational formulas, the resonant frequency will be the same as the one obtained by the mode-matching method.**

***Index Terms*—Cavity resonators, dielectric resonators, eigenvalue problem, mode-matching method, reaction, variational formulation.**

I. INTRODUCTION

REALIZATION of high dielectric-constant and high quality-factor ceramics make it possible to achieve very stable and small-size resonators and filters in microwave/millimeter-wave technology. To achieve good mechanical stability, cylindrical dielectric resonators (DRs) are loaded in rectangular enclosures for dual-mode filters [1]. Cylindrical DRs inside cylindrical enclosures have been successfully analyzed by the classic mode-matching method [2], [3]. However, this approach cannot be used to analyze a cylindrical DR loaded in a rectangular box. The reason lies in the fact that the boundary conditions do not coincide with constant coordinate surfaces. Unlike open structures, e.g., [4], using integral-equation formulation of DRs inside rectangular enclosures becomes too complicated because of a lack of a closed-form Green's function and slowly convergent nature of other existing forms. Scattering by a cylindrical post of complex permittivity in a waveguide was investigated before by the point-matching method [5] and a lossy dielectric post extending the full height of a rectangular waveguide was considered by Gesche and Löchel [6]. In [6], the Bessel–Fourier series is used to obtain overlap integrals. Following the same approach, a generalized Bessel–Fourier series is used to analyze more general DR structures [7].

Recently, by generalizing the reaction concept in electromagnetic theory, we have developed unrestricted and nonunique variational formulas for cavity resonators, which relax any specific boundary condition on the trial fields [8]. We have observed that when a trial field obtained by the mode-matching method is used in a particular form of the variational expressions, the resonant frequency does not change. This behavior states that this solution is a stationary point of a variational expression and it should be expected since mode-matching method is equivalent to the Galerkin approach. We are then led to generalize the mode-matching method to nonorthogonal and free-boundary cases where a set of basis functions used for the field expansion inside a cavity resonator need not to be orthogonal or satisfy any specific boundary conditions. Based on this new formulation, cylindrical DRs inside rectangular cavities can be analyzed by expanding the electromagnetic field in terms of a not necessarily orthogonal set of basis functions. Since the basis functions are not required to satisfy any specific boundary conditions, this approach can potentially handle various aperture or tuning mechanisms within a cavity resonator.

This paper is organized as follows. Details of this new formulation are given in Section II. Section III is devoted to numerical results, and conclusions are summarized in Section IV.

II. BASIC FORMULATION

The materials presented in this section are based on Part I of this paper [8] and it is assumed that the reader already reviewed it. The variational formulas derived in [8] put no limitation on the class of the trial fields. In those formulas, since the frequency is explicitly shown as a function of the trial field, we call those formulas *explicit* ones. This fact is the direct consequence of having $\langle \mathbf{c}+p\mathbf{e}, \mathbf{c}+p\mathbf{e}\rangle_\alpha$ proportional to $(j\omega_r)^n D_\alpha(p)+N_\alpha(p)$, which, in turn, is the consequence of inclusion of the volume

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A. M. Shams-Zadeh-Amiri was with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, Canada N2L 3G1. He is now with PhotoNami Inc., Richmond Hill, ON, Canada L4B 1E4.

S. Safavi-Naeini and S. K. Chaudhuri are with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, Canada N2L 3G1 (e-mail: safavi@maxwell.uwaterloo.ca).

R. Sabry is with Ericsson Canada Inc., Mississauga, ON, Canada L4W 5E3 and also with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, Canada N2L 3G1.

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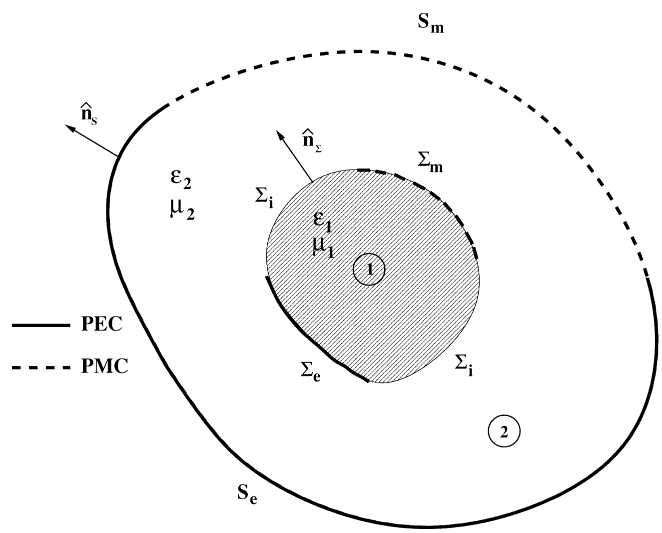

Fig. 1. Arbitrary cavity resonator.

sources in the formulation given in [8]. Applicability of the explicit formulas is limited to simple structures and their use in more complicated structures is less practical because of the difficulty of finding the trial fields. On the other hand, there exists a class of trial fields, which do not need any volume sources to be supported. This class is a natural choice because the exact resonant field is a source-free one. Since no volume current is needed for these trial fields, the volume integrals will not necessarily appear in the expression for $\langle \mathbf{a}, \mathbf{a} \rangle_\alpha$. Therefore, the resonant frequency cannot be explicitly expressed in terms of the trial fields and Harrington's approach [9, pp. 340–345] is not applicable. However, by using the Rayleigh–Ritz method, the resonant frequencies can be obtained by a matrix equation and seeking the zeros of the determinant of a matrix. Of course, for this class of trial fields, the term $\int_V \mathbf{E}^a \cdot \mathbf{J}^a \, dv - \int_V \mathbf{H}^a \cdot \mathbf{M}^a \, dv$ identically vanishes, and by adding this term to $\langle \mathbf{a}, \mathbf{a} \rangle_\alpha$ and noting that the derivative of an identically zero term with respect to the variational parameter p also vanishes, one may see that the formulas in [8] are also valid for this type of trial fields. We will address this issue shortly.

Let us consider the cavity resonator shown in Fig. 1. Assume that $\mathbf{E}^a$ and $\mathbf{H}^a$ are arbitrary vector fields defined within the cavity such that at some resonant frequency ω_r they satisfy homogeneous Maxwell's equations $\nabla \times \mathbf{E}^a_{1,2} = -j\omega_r \mu_{1,2} \mathbf{H}^a_{1,2}$ and $\nabla \times \mathbf{H}^a_{1,2} = j\omega_r \epsilon_{1,2} \mathbf{E}^a_{1,2}$. Including the necessary surface currents to support the discontinuities across the boundaries [8], we get

$$\begin{aligned}\langle \mathbf{a}, \mathbf{a} \rangle_\alpha = &\int_{\Sigma_m} \left(\mathbf{E}^a_1 \times \mathbf{H}^a_1 - \mathbf{E}^a_2 \times \mathbf{H}^a_2 \right) \cdot \hat{\mathbf{n}}_\Sigma \, ds \\ &- \int_{\Sigma_e} \left(\mathbf{E}^a_1 \times \mathbf{H}^a_1 - \mathbf{E}^a_2 \times \mathbf{H}^a_2 \right) \cdot \hat{\mathbf{n}}_\Sigma \, ds \\ &+ \int_{S_m} \mathbf{E}^a_2 \times \mathbf{H}^a_2 \cdot \hat{\mathbf{n}}_S \, ds - \int_{S_e} \mathbf{E}^a_2 \times \mathbf{H}^a_2 \cdot \hat{\mathbf{n}}_S \, ds \\ &+ \int_{\Sigma_i} \left(\mathbf{E}^a_2 \times \mathbf{H}^a_1 - \mathbf{E}^a_1 \times \mathbf{H}^a_2 \right) \cdot \hat{\mathbf{n}}_\Sigma \, ds \\ &+ \int_{\Sigma_i} \left[2\alpha(\sigma) - 1 \right] \left(\mathbf{E}^a_1 - \mathbf{E}^a_2 \right) \\ &\times \left(\mathbf{H}^a_1 - \mathbf{H}^a_2 \right) \cdot \hat{\mathbf{n}}_\Sigma \, ds \end{aligned} \tag{1}$$

where Σ_e and Σ_m are parts of the interior boundary made of perfect electric and perfect magnetic conductors, respectively. Σ_i is just the interface between the two dielectric materials and S_e and S_m, respectively, are parts of the cavity walls made of perfect electric and perfect magnetic conductors. According to the fundamental lemma in [8], the above expression is stationary about the exact resonant field distribution. In fact, if we write $\mathbf{E}^a_{1,2} = \mathbf{E}^c_{1,2} + p\mathbf{E}^e_{1,2}$ and $\mathbf{H}^a_{1,2} = \mathbf{H}^c_{1,2} + p\mathbf{H}^e_{1,2}$, where $\mathbf{E}^c_{1,2}$ and $\mathbf{H}^c_{1,2}$ are the exact resonant field distributions and $\mathbf{E}^e_{1,2}$ and $\mathbf{H}^e_{1,2}$ satisfy homogeneous Maxwell's equations at the same resonant frequency, it can be shown that

$$\left[\frac{d \langle \mathbf{c} + p\mathbf{e}, \mathbf{c} + p\mathbf{e} \rangle_\alpha}{dp} \right]_{p=0} = \langle \mathbf{c}, \mathbf{e} \rangle_\alpha = 0 \tag{2}$$

which indicates the stationary character of (1). Equation (1) can be converted to an expression in the $\mathbf{E}$- or $\mathbf{H}$-field form if one obtains the $\mathbf{H}$-field in terms of the $\mathbf{E}$-field or vice versa. However, the resulting equations do not have any advantage over (1). As can be read from (1), the frequency is not shown explicitly in the expression for $\langle \mathbf{a}, \mathbf{a} \rangle_\alpha$. Hence, we call it the *implicit* formula. However, by using the stationary character of (1) about the exact resonant field, one may apply the Rayleigh–Ritz method to expand the fields in terms of some basis functions and obtain the resonant frequency and the unknown expansion coefficients by a matrix equation.

Equation (1), which is obtained based on the assumption of source-free trial fields, can be reduced to those explicit ones in [8]. To this end, we add the following expression to the right-hand side of (1):

$$\begin{aligned} &U_{eh}(\mathbf{E}^a, \mathbf{H}^a) \\ &\quad \triangleq \int_{V_1+V_2} \mathbf{E}^a_{1,2} \cdot \mathbf{J}^a_{1,2} \, ds - \int_{V_1+V_2} \mathbf{H}^a_{1,2} \cdot \mathbf{M}^a_{1,2} \, ds \\ &\quad = \int_{V_1+V_2} \mathbf{E}^a_{1,2} \cdot \left(\nabla \times \mathbf{H}^a_{1,2} - j\omega_r \epsilon_{1,2} \mathbf{E}^a_{1,2} \right) ds \\ &\qquad - \int_{V_1+V_2} \mathbf{H}^a_{1,2} \cdot \left(-\nabla \times \mathbf{E}^a_{1,2} - j\omega_r \mu_{1,2} \mathbf{H}^a_{1,2} \right) ds. \end{aligned} \tag{3}$$

Since for source-free trial fields $\mathbf{J}^a_{1,2} = \mathbf{M}^a_{1,2} = 0, U_{eh}(\mathbf{E}^a, \mathbf{H}^a) \equiv 0$. Moreover, if $\mathbf{E}^e$ and $\mathbf{H}^e$ satisfy homogeneous Maxwell's equations at the same resonant frequency ω_r and we define $U(p) \triangleq U_{eh}(\mathbf{E}^c + p\mathbf{E}^e, \mathbf{H}^c + p\mathbf{H}^e)$, then $U(p) \equiv 0$ and, therefore, $[dU/dp]_{p=0} = 0$. More precisely, we have

$$\begin{aligned} [dU/dp]_{p=0} = &\int_{V_1+V_2} \left(\mathbf{E}^e_{1,2} \cdot \mathbf{J}^c_{1,2} + \mathbf{E}^c_{1,2} \cdot \mathbf{J}^e_{1,2} \right) dv \\ &- \int_{V_1+V_2} \left(\mathbf{H}^e_{1,2} \cdot \mathbf{M}^c_{1,2} + \mathbf{H}^c_{1,2} \cdot \mathbf{M}^e_{1,2} \right) dv \\ = &\, 0 \end{aligned} \tag{4}$$

where we have used $\mathbf{J}^c_{1,2} = \mathbf{J}^e_{1,2} = \mathbf{M}^c_{1,2} = \mathbf{M}^e_{1,2} = 0$. Now the mathematical expression of $\langle \mathbf{a}, \mathbf{a} \rangle_\alpha + U_{eh}(\mathbf{E}^a, \mathbf{H}^a)$ is the same as [8, eq. (43)]. Moreover, if for a fixed $\mathbf{E}^e$ and

$\mathbf{H}^e$ satisfying homogeneous Maxwell's equations at the same resonant frequency ω_r, one defines

$$g_\alpha(p) = \langle \mathbf{c} + p\mathbf{e}, \mathbf{c} + p\mathbf{e}\rangle_\alpha + U(p) \tag{5}$$

then we have $g_\alpha(0) = g'_\alpha(0) = 0$ which means that [8, eq. (46)] is stationary about the correct resonant field for the trial fields having no volume sources at the resonant frequency.

Let us consider the following equation:

$$\omega_\alpha^2 = \frac{N_\alpha^e(\mathbf{E}^a)}{D_\alpha^e(\mathbf{E}^a)} \tag{6}$$

where $N_\alpha^e(\mathbf{E}^a)$ and $D_\alpha^e(\mathbf{E}^a)$ are given, respectively, in [8, eq. (34) and (35)]. By the same argument, it can be shown that (6) is variational about the correct resonant field for the trial fields without any volume sources at the resonant frequency if one substitutes $\mathbf{H}_{1,2}^a$ with $j\omega_r^{-1}\mu_{1,2}^{-1}\nabla\times\mathbf{E}_{1,2}^a$ in (1) and adds the following expression to the right-hand side of (1):

$$\int_{V_1+V_2} \mathbf{E}_{1,2}^a\cdot\mathbf{J}_{1,2}^a\,ds = \int_{V_1+V_2} \mathbf{E}_{1,2}^a\cdot\left(-j\omega_r\epsilon\mathbf{E}_{1,2}^a + j\omega_r^{-1}\nabla \times\mu_{1,2}^{-1}\nabla\times\mathbf{E}_{1,2}^a\right)dv.$$

Finally, the $\mathbf{H}$-field formulation [8, eq. (40)] is also variational about the correct resonant field if one uses a trial field without any volume source at the resonant frequency.

Let us turn to (1). As mentioned earlier, the resonant frequency does not appear explicitly in that equation and the Rayleigh–Ritz method is the only way to deal with the resonant frequency. Under a special case where $\alpha(\sigma)\equiv 1/2$ (see [8] for the definition of $\alpha(\sigma)$), the last term on the right-hand side of (1) vanishes and we have

$$\begin{aligned}\langle\mathbf{a},\mathbf{a}\rangle_{1/2} &= \int_{\Sigma_m}\left(\mathbf{E}_1^a\times\mathbf{H}_1^a - \mathbf{E}_2^a\times\mathbf{H}_2^a\right)\cdot\hat{\mathbf{n}}_\Sigma\,ds\\ &\quad-\int_{\Sigma_e}\left(\mathbf{E}_1^a\times\mathbf{H}_1^a - \mathbf{E}_2^a\times\mathbf{H}_2^a\right)\cdot\hat{\mathbf{n}}_\Sigma\,ds\\ &\quad+\int_{S_m}\mathbf{E}_2^a\times\mathbf{H}_2^a\cdot\hat{\mathbf{n}}_S\,ds - \int_{S_e}\mathbf{E}_2^a\times\mathbf{H}_2^a\cdot\hat{\mathbf{n}}_S\,ds\\ &\quad+\int_{\Sigma_i}\left(\mathbf{E}_2^a\times\mathbf{H}_1^a - \mathbf{E}_1^a\times\mathbf{H}_2^a\right)\cdot\hat{\mathbf{n}}_\Sigma\,ds.\end{aligned} \tag{7}$$

Now, we intend to show that by applying the Rayleigh–Ritz method to (7) leads to a systematic method for deriving a system of homogeneous linear equations, characterizing the cavity resonator, which we call the *generalized mode-matching method* (GMMM). As a special case, the equations obtained by this generalized formulation reduce to those obtained by the conventional mode-matching method if the latter is applicable to a cavity resonator. To this end, suppose that the correct resonant field inside the cavity shown in Fig. 1 can be expanded in terms of a not necessarily orthogonal set of basis functions $\hat{\mathbf{E}}_k^{(1,2)}$ and $\hat{\mathbf{H}}_k^{(1,2)}$ as follows:

$$\begin{aligned}\begin{bmatrix}\mathbf{E}_1^c(\mathbf{r},\omega_r)\\ \mathbf{H}_1^c(\mathbf{r},\omega_r)\end{bmatrix} &= \begin{bmatrix}\sum_{m=1}^\infty A_m\hat{\mathbf{E}}_m^{(1)}(\mathbf{r},\omega_r)\\ \sum_{m=1}^\infty A_m\hat{\mathbf{H}}_m^{(1)}(\mathbf{r},\omega_r)\end{bmatrix}\\ \begin{bmatrix}\mathbf{E}_2^c(\mathbf{r},\omega_r)\\ \mathbf{H}_2^c(\mathbf{r},\omega_r)\end{bmatrix} &= \begin{bmatrix}\sum_{n=1}^\infty B_n\hat{\mathbf{E}}_n^{(2)}(\mathbf{r},\omega_r)\\ \sum_{n=1}^\infty B_n\hat{\mathbf{H}}_n^{(2)}(\mathbf{r},\omega_r)\end{bmatrix}\end{aligned} \tag{8}$$

where $\hat{\mathbf{E}}_k^{(1,2)}$ and $\hat{\mathbf{H}}_k^{(1,2)}$ satisfy homogeneous Maxwell's equations at the resonant frequency ω_r. A_m's and B_n's are the exact yet unknown expansion coefficients. Suppose that one of these coefficients, e.g., A_u, is changed to $A_u + p$. The correct field distribution in region 1 with this change of coefficient reduces to the approximate one as

$$\begin{bmatrix}\mathbf{E}_1^a(\mathbf{r},\omega_r)\\ \mathbf{H}_1^a(\mathbf{r},\omega_r)\end{bmatrix} = \begin{bmatrix}\mathbf{E}_1^c(\mathbf{r},\omega_r) + p\hat{\mathbf{E}}_u^{(1)}(\mathbf{r},\omega_r)\\ \mathbf{H}_1^c(\mathbf{r},\omega_r) + p\hat{\mathbf{H}}_u^{(1)}(\mathbf{r},\omega_r)\end{bmatrix} \tag{9}$$

whereas, in region 2, it remains unchanged and is the same as the correct resonant field distribution. Therefore, by defining the error field $\mathbf{e}$ as

$$\begin{bmatrix}\mathbf{E}_1^e(\mathbf{r},\omega_r)\\ \mathbf{H}_1^e(\mathbf{r},\omega_r)\end{bmatrix} = \begin{bmatrix}\hat{\mathbf{E}}_u^{(1)}(\mathbf{r},\omega_r)\\ \hat{\mathbf{H}}_u^{(1)}(\mathbf{r},\omega_r)\end{bmatrix} \quad \begin{bmatrix}\mathbf{E}_2^e(\mathbf{r},\omega_r)\\ \mathbf{H}_2^e(\mathbf{r},\omega_r)\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \tag{10}$$

and using $\mathbf{a} = \mathbf{c} + p\mathbf{e}$ from (7), we get

$$\begin{aligned}&\left[\frac{d\langle\mathbf{c}+p\mathbf{e},\mathbf{c}+p\mathbf{e}\rangle_{1/2}}{dp}\right]_{p=0}\\ &= \int_{\Sigma_m}\mathbf{E}_1^c(\mathbf{r},\omega_r)\times\hat{\mathbf{H}}_u^{(1)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}_\Sigma\,ds\\ &\quad-\int_{\Sigma_e}\hat{\mathbf{E}}_u^{(1)}(\mathbf{r},\omega_r)\times\mathbf{H}_1^c(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}_\Sigma\,ds\\ &\quad+\int_{\Sigma_i}\left[\mathbf{E}_2^c(\mathbf{r},\omega_r)\times\hat{\mathbf{H}}_u^{(1)}(\mathbf{r},\omega_r) - \hat{\mathbf{E}}_u^{(1)}(\mathbf{r},\omega_r)\times\mathbf{H}_2^c\right]\\ &\quad\cdot\hat{\mathbf{n}}_\Sigma\,ds\\ &= \langle\mathbf{c},\mathbf{e}\rangle_{1/2}\\ &= 0\end{aligned} \tag{11}$$

where we have used the fact that the correct resonant field $\mathbf{c}$ satisfies all boundary conditions. The above result is expected because $[(d/dp)\langle\mathbf{c}+p\mathbf{e},\mathbf{c}+p\mathbf{e}\rangle_{1/2}]_{p=0} = 0$. Using (8) to substitute for $\mathbf{E}_{1,2}^c$ and $\mathbf{H}_{1,2}^c$ in (11), one may see that

$$\left[\frac{d\langle\mathbf{c}+p\mathbf{e},\mathbf{c}+p\mathbf{e}\rangle_{1/2}}{dp}\right]_{p=0} = \frac{\partial\langle\mathbf{c},\mathbf{c}\rangle_{1/2}}{\partial A_u} \triangleq \left[\frac{\partial\langle\mathbf{a},\mathbf{a}\rangle_{1/2}}{\partial A_u}\right]_{p=0} \tag{12}$$

where, as indicated, $\langle\mathbf{c},\mathbf{c}\rangle_{1/2}$ can be obtained from (7) by substituting $\mathbf{E}^c$ and $\mathbf{H}^c$ for $\mathbf{E}^a$ and $\mathbf{H}^a$, respectively, i.e.,

$$\begin{aligned}\langle\mathbf{c},\mathbf{c}\rangle_{1/2} &= \int_{\Sigma_m}\left(\mathbf{E}_1^c\times\mathbf{H}_1^c - \mathbf{E}_2^c\times\mathbf{H}_2^c\right)\cdot\hat{\mathbf{n}}_\Sigma\,ds\\ &\quad-\int_{\Sigma_e}\left(\mathbf{E}_1^c\times\mathbf{H}_1^c - \mathbf{E}_2^c\times\mathbf{H}_2^c\right)\cdot\hat{\mathbf{n}}_\Sigma\,ds\\ &\quad+\int_{S_m}\mathbf{E}_2^c\times\mathbf{H}_2^c\cdot\hat{\mathbf{n}}_S\,ds - \int_{S_e}\mathbf{E}_2^c\times\mathbf{H}_2^c\cdot\hat{\mathbf{n}}_S\,ds\\ &\quad+\int_{\Sigma_i}\left(\mathbf{E}_2^c\times\mathbf{H}_1^c - \mathbf{E}_1^c\times\mathbf{H}_2^c\right)\cdot\hat{\mathbf{n}}_\Sigma\,ds.\end{aligned} \tag{13}$$

Equation (12), which is a linear equation in A_m's and B_n's, is the mathematical statement of the so-called Rayleigh–Ritz method. The systematic approach that bypasses using (13) for obtaining $(\partial/\partial A_u)\langle \mathbf{c},\mathbf{c}\rangle_{1/2}$ leads to the GMMM. To this end, by using (8) to substitute for $\mathbf{E}^c$ and $\mathbf{H}^c$ in (13) and interchanging the summation with integral, we have

$$\begin{aligned}\frac{\partial\langle \mathbf{c},\mathbf{c}\rangle_{1/2}}{\partial A_u} =& \sum_{m=1}^{\infty} A_m \int_{\Sigma_m} \hat{\mathbf{E}}_m^{(1)}(\mathbf{r},\omega_r)\times\hat{\mathbf{H}}_u^{(1)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}_\Sigma\, ds\\ &-\sum_{m=1}^{\infty} A_m \int_{\Sigma_e} \hat{\mathbf{E}}_u^{(1)}(\mathbf{r},\omega_r)\times\hat{\mathbf{H}}_m^{(1)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}_\Sigma\, ds\\ &+\sum_{n=1}^{\infty} B_n \int_{\Sigma_i} \hat{\mathbf{E}}_n^{(2)}(\mathbf{r},\omega_r)\times\hat{\mathbf{H}}_u^{(1)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}_\Sigma\, ds\\ &-\sum_{n=1}^{\infty} B_n \int_{\Sigma_i} \hat{\mathbf{E}}_u^{(1)}(\mathbf{r},\omega_r)\times\hat{\mathbf{H}}_n^{(2)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}_\Sigma\, ds\\ =&\ 0. \end{aligned}\tag{14}$$

Let us write the boundary conditions on $\Sigma=\Sigma_i+\Sigma_e+\Sigma_m$ as shown in the following:

$$\begin{aligned}&\hat{\mathbf{n}}_\Sigma\times\left[\mathbf{E}_1^c(\mathbf{r},\omega_r)\right]\\ &=\begin{cases}0, \quad \text{if } \mathbf{r}\in\Sigma_e^1\\ \hat{\mathbf{n}}_\Sigma\times[\mathbf{E}_2^c(\mathbf{r},\omega_r)]=\hat{\mathbf{n}}_\Sigma\times\left[\sum_{n=1}^{\infty}B_n\hat{\mathbf{E}}_n^{(2)}(\mathbf{r},\omega_r)\right], \\ \qquad \text{if } \mathbf{r}\in\Sigma_i\\ \hat{\mathbf{n}}_\Sigma\times[\mathbf{E}_1^c(\mathbf{r},\omega_r)]=\hat{\mathbf{n}}_\Sigma\times\left[\sum_{m=1}^{\infty}A_m\hat{\mathbf{E}}_m^{(1)}(\mathbf{r},\omega_r)\right], \\ \qquad \text{if } \mathbf{r}\in\Sigma_m^1\end{cases}\end{aligned}\tag{15}$$

$$\begin{aligned}&\left[\mathbf{H}_1^c(\mathbf{r},\omega_r)\right]\times\hat{\mathbf{n}}_\Sigma\\ &=\begin{cases}[\mathbf{H}_1^c(\mathbf{r},\omega_r)]\times\hat{\mathbf{n}}_\Sigma=\left[\sum_{m=1}^{\infty}A_m\hat{\mathbf{H}}_m^{(1)}(\mathbf{r},\omega_r)\right]\times\hat{\mathbf{n}}_\Sigma, \\ \qquad \text{if } \mathbf{r}\in\Sigma_e^1\\ [\mathbf{H}_2^c(\mathbf{r},\omega_r)]\times\hat{\mathbf{n}}_\Sigma=\left[\sum_{n=1}^{\infty}B_n\hat{\mathbf{H}}_n^{(2)}(\mathbf{r},\omega_r)\right]\times\hat{\mathbf{n}}_\Sigma, \\ \qquad \text{if } \mathbf{r}\in\Sigma_i\\ 0, \quad \text{if } \mathbf{r}\in\Sigma_m^1\end{cases}\end{aligned}\tag{16}$$

where Σ_e^1 and Σ_m^1 are those points on Σ_e and Σ_m, respectively, immediately inside region 1. The key idea of writing (15) is equating the tangential component of the electric field in each region to itself on the magnetic walls and to the corresponding tangential component of the electric field in the neighboring region at the interface. Of course, the tangential components of the exact electric field vanish on the perfect electric wall. Equation (16) is written in a dual fashion for the magnetic field. By dot multiplying (15) with $\hat{\mathbf{H}}_u^{(1)}$ and (16) with $\hat{\mathbf{E}}_u^{(1)}$ and integrating over Σ, we obtain

$$\begin{aligned}&\sum_{m=1}^{\infty} A_m \int_{\Sigma} \hat{\mathbf{E}}_m^{(1)}(\mathbf{r},\omega_r)\times\hat{\mathbf{H}}_u^{(1)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}_\Sigma\, ds\\ &\quad=\sum_{n=1}^{\infty} B_n \int_{\Sigma_i} \hat{\mathbf{E}}_n^{(2)}(\mathbf{r},\omega_r)\times\hat{\mathbf{H}}_u^{(1)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}_\Sigma\, ds\\ &\qquad+\sum_{m=1}^{\infty} A_m \int_{\Sigma_m} \hat{\mathbf{E}}_m^{(1)}(\mathbf{r},\omega_r)\times\hat{\mathbf{H}}_u^{(1)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}_\Sigma\, ds\end{aligned}\tag{17}$$

$$\begin{aligned}&\sum_{m=1}^{\infty} A_m \int_{\Sigma} \hat{\mathbf{E}}_u^{(1)}(\mathbf{r},\omega_r)\times\hat{\mathbf{H}}_m^{(1)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}_\Sigma\, ds\\ &\quad=\sum_{m=1}^{\infty} A_m \int_{\Sigma_e} \hat{\mathbf{E}}_u^{(1)}(\mathbf{r},\omega_r)\times\hat{\mathbf{H}}_m^{(1)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}_\Sigma\, ds\\ &\qquad+\sum_{n=1}^{\infty} B_n \int_{\Sigma_i} \hat{\mathbf{E}}_u^{(1)}(\mathbf{r},\omega_r)\times\hat{\mathbf{H}}_n^{(2)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}_\Sigma\, ds\end{aligned}\tag{18}$$

where we have used (8) to substitute for $\mathbf{E}_1^c$ and $\mathbf{H}_1^c$. In the conventional mode-matching method, where the basis functions or the so-called modes are orthogonal on Σ, the left-hand sides of (17) and (18) are equal to $A_u\int_\Sigma \hat{\mathbf{E}}_u^{(1)}(\mathbf{r},\omega_r)\times\hat{\mathbf{H}}_u^{(1)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}_\Sigma\, ds$ because $\int_\Sigma \hat{\mathbf{E}}_u^{(1)}(\mathbf{r},\omega_r)\times\hat{\mathbf{H}}_v^{(1)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}_\Sigma\, ds=0$ if $u\neq v$. Therefore, by equating the right-hand sides of (17) and (18), we end up with (14). For the general case where the basis functions are not orthogonal, it is not clear that the left-hand sides of (17) and (18) are equal. However, by subtracting both sides of (18) from those of (17), the right-hand side of the resulting equation is simply $\langle \mathbf{c},\mathbf{e}\rangle_{1/2}=0$, where $\mathbf{e}$ is given by (10). Therefore, the left-hand sides of (17) and (18) must be equal. From the above considerations, to obtain a system of homogeneous linear equations in A_m's and B_n's, one may start with (17) and (18) and treat it as if the basis functions are orthogonal on Σ and equate the right-hand sides of the equations.

Using a similar argument, by defining the error field $\mathbf{e}$ as

$$\begin{bmatrix}\mathbf{E}_1^e(\mathbf{r},\omega_r)\\ \mathbf{H}_1^e(\mathbf{r},\omega_r)\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}\quad\begin{bmatrix}\mathbf{E}_2^e(\mathbf{r},\omega_r)\\ \mathbf{H}_2^e(\mathbf{r},\omega_r)\end{bmatrix}=\begin{bmatrix}\hat{\mathbf{E}}_v^{(2)}(\mathbf{r},\omega_r)\\ \hat{\mathbf{H}}_v^{(2)}(\mathbf{r},\omega_r)\end{bmatrix}\tag{19}$$

and $\mathbf{a}=\mathbf{c}+p\mathbf{e}$, one may show that

$$\begin{aligned}0&=\left[\frac{d\langle \mathbf{c}+p\mathbf{e},\mathbf{c}+p\mathbf{e}\rangle_{1/2}}{dp}\right]_{p=0}\\ &=\frac{\partial\langle \mathbf{c},\mathbf{c}\rangle_{1/2}}{\partial B_v}\\ &\triangleq\left[\frac{\partial\langle \mathbf{a},\mathbf{a}\rangle_{1/2}}{\partial B_v}\right]_{p=0}.\end{aligned}\tag{20}$$

To obtain $(\partial/\partial B_v)\langle \mathbf{c},\mathbf{c}\rangle_{1/2}$ in a systematic way, we write

$$\begin{aligned}&\hat{\mathbf{n}}\times[\mathbf{E}_2^c(\mathbf{r},\omega_r)]\\ &=\begin{cases}0, \quad \text{if } \mathbf{r}\in\Sigma_e^2 \text{ and } S_e\\ \hat{\mathbf{n}}\times[\mathbf{E}_1^c(\mathbf{r},\omega_r)]=\hat{\mathbf{n}}\times\left[\sum_{m=1}^{\infty}A_m\hat{\mathbf{E}}_m^{(1)}(\mathbf{r},\omega_r)\right], \\ \qquad \text{if } \mathbf{r}\in\Sigma_i\\ \hat{\mathbf{n}}\times[\mathbf{E}_2^c(\mathbf{r},\omega_r)]=\hat{\mathbf{n}}\times\left[\sum_{n=1}^{\infty}B_n\hat{\mathbf{E}}_n^{(2)}(\mathbf{r},\omega_r)\right], \\ \qquad \text{if } \mathbf{r}\in\Sigma_m^2 \text{ and } S_m\end{cases}\end{aligned}\tag{21}$$

$$\begin{aligned}&[\mathbf{H}_2^c(\mathbf{r},\omega_r)]\times\hat{\mathbf{n}}\\ &=\begin{cases}[\mathbf{H}_2^c(\mathbf{r},\omega_r)]\times\hat{\mathbf{n}}=\left[\sum_{n=1}^{\infty}B_n\hat{\mathbf{H}}_n^{(2)}(\mathbf{r},\omega_r)\right]\times\hat{\mathbf{n}}, \\ \qquad \text{if } \mathbf{r}\in\Sigma_e^2 \text{ and } S_e\\ [\mathbf{H}_1^c(\mathbf{r},\omega_r)]\times\hat{\mathbf{n}}=\left[\sum_{m=1}^{\infty}A_m\hat{\mathbf{H}}_m^{(1)}(\mathbf{r},\omega_r)\right]\times\hat{\mathbf{n}}, \\ \qquad \text{if } \mathbf{r}\in\Sigma_i\\ 0, \quad \text{if } \mathbf{r}\in\Sigma_m^2 \text{ and } S_m\end{cases}\end{aligned}\tag{22}$$

where Σ_e^2 and Σ_m^2 are the points on Σ_e and Σ_m, respectively, immediately inside region 2. To be consistent, we have directed the unit normal to the boundary of region 2 out of that region. Therefore, $\hat{\mathbf{n}} = -\hat{\mathbf{n}}_\Sigma$ on Σ and $\hat{\mathbf{n}} = \hat{\mathbf{n}}_S$ on $S = S_e + S_m$. By dot multiplying (21) with $\hat{\mathbf{H}}_v^{(2)}$ and (22) with $\hat{\mathbf{E}}_v^{(2)}$ and integrating over the surface $\Sigma + S$, we obtain

$$\begin{aligned}\sum_{n=1}^{\infty} B_n \int_{\Sigma+S} \hat{\mathbf{E}}_n^{(2)}(\mathbf{r},\omega_r) \times \hat{\mathbf{H}}_v^{(2)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}\,ds \\ = \sum_{m=1}^{\infty} A_m \int_{\Sigma_i} \hat{\mathbf{E}}_m^{(1)}(\mathbf{r},\omega_r) \times \hat{\mathbf{H}}_v^{(2)}(\mathbf{r},\omega_r)\cdot(-\hat{\mathbf{n}}_\Sigma)\,ds \\ + \sum_{n=1}^{\infty} B_n \int_{\Sigma_m+S_m} \hat{\mathbf{E}}_n^{(2)}(\mathbf{r},\omega_r) \times \hat{\mathbf{H}}_v^{(2)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}\,ds\end{aligned} \tag{23}$$

$$\begin{aligned}\sum_{n=1}^{\infty} B_n \int_{\Sigma+S} \hat{\mathbf{E}}_v^{(2)}(\mathbf{r},\omega_r) \times \hat{\mathbf{H}}_n^{(2)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}\,ds \\ = \sum_{n=1}^{\infty} B_n \int_{\Sigma_e+S_e} \hat{\mathbf{E}}_v^{(2)}(\mathbf{r},\omega_r) \times \hat{\mathbf{H}}_n^{(2)}(\mathbf{r},\omega_r)\cdot\hat{\mathbf{n}}\,ds \\ + \sum_{m=1}^{\infty} A_m \int_{\Sigma_i} \hat{\mathbf{E}}_v^{(2)}(\mathbf{r},\omega_r) \times \hat{\mathbf{H}}_m^{(1)}(\mathbf{r},\omega_r)\cdot(-\hat{\mathbf{n}}_\Sigma)\,ds\end{aligned} \tag{24}$$

where we have used (8) to substitute for $\mathbf{E}_2^c$ and $\mathbf{H}_2^c$. In the conventional mode-matching method, the basis functions are orthogonal on the surface Σ and $\hat{\mathbf{n}}_S \times \hat{\mathbf{E}}_n$ and $\hat{\mathbf{n}}_S \times \hat{\mathbf{H}}_n$ vanish on S_e and S_m, respectively. In either case, the left-hand sides of (23) and (24) are equal and by subtracting the right-hand side of (24) from that of (23), one may obtain the expression for $(\partial/\partial B_v)\langle \mathbf{c},\mathbf{c}\rangle_{1/2}$. For the cases where the basis functions do not satisfy the above constraints, the left-hand sides of (23) and (24) are also equal. This is because by subtracting the right-hand side of (24) from that of (23), one obtains the mathematical expression of $\langle \mathbf{c},\mathbf{e}\rangle_{1/2}$ where the error field is given by (19). Therefore, to obtain another set of linear equations in A_m's and B_n's, one may start with (23) and (24) and equate the right-hand sides of those equations.

The above systematic method for obtaining a system of homogeneous linear equations in unknown coefficients A_m's and B_n's is based on the variational principle. In practice, like the conventional mode-matching method, we express the unknown field in each region as a linear combination of a finite number of basis functions, which satisfy homogeneous Maxwell's equations at an arbitrary frequency and set up the system of homogeneous linear equations, as explained above. Of course, since these linear equations hold only at the resonant frequency of the cavity, by seeking the zeros of the determinant of the coefficient matrix, one may obtain the resonant frequency and the unknown expansion coefficients within a multiplicative constant.

Since the conventional mode-matching method is a special case of this generalized formulation, one may see that the former is also based on the variational principle. In the Appendix, we will prove that this is, in fact, the case and show that in a modal structure if the field distribution inside a cavity resonator obtained based on the conventional mode-matching method is used in any variational formula with $\alpha = 1/2$, the frequency will be the same as that obtained by the mode-matching method. In other words, in a modal structure, one cannot improve the accuracy of the resonant frequency obtained based on the mode-matching method by a variational formula with $\alpha = 1/2$.

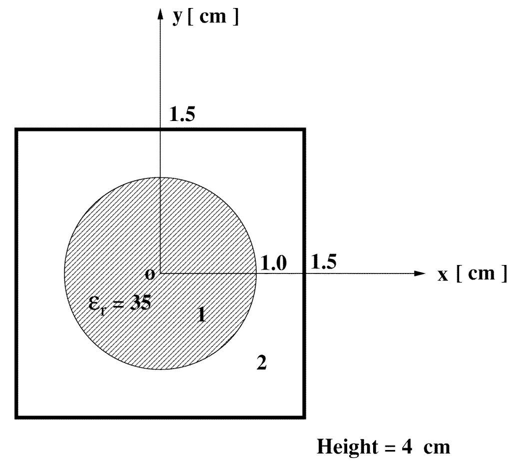

Fig. 2. Cross section of a rectangular cavity resonator with a cylindrical dielectric rod inside it.

III. Numerical Results

In this section, we apply the formulation developed in the preceding section to some physical structures. In the first example, we use the GMMM to obtain the resonant frequency of a rectangular cavity containing a cylindrical dielectric rod, which extends the full height of the cavity. To this end, we consider two separate regions inside the whole cavity, as illustrated in Fig. 2. Region 1 is the set of points inside the cylindrical rod and the rest of the cavity is considered as region 2. We expand the field in region 1 in terms of the cylindrical harmonic obtained by the Hertzian potential $\mathbf{\Pi}_{1_{00}}^e = \Psi_{1_{00}}^e \hat{\mathbf{z}} = J_o(k_\rho \rho)\hat{\mathbf{z}}$, where $k_\rho = \sqrt{\epsilon_r} k_o$. The field inside region 2 is expanded in terms of the basis functions obtained by $\mathbf{\Pi}_{2_{n0}}^e = \Psi_{2_{n0}}^e \hat{\mathbf{z}} = \sin[\alpha_{n0}(x+a)]\sin[n\pi(y+a)/2a]\hat{\mathbf{z}}$, $(n = 1, 2, \ldots)$, where $\alpha_{n0} = \sqrt{k_o^2 - (n\pi/2a)^2}$ and $a = 1.5$ cm. It should be emphasized that the basis functions in region 2 do not satisfy the boundary conditions at $x = 1.5$ cm. Recall that the electric and magnetic fields can be obtained in terms of the Hertzian potential function as follows:

$$\begin{aligned}\mathbf{H}^e &= -j\omega\epsilon_o \hat{\mathbf{z}} \times \nabla\Psi^e \\ \mathbf{E}^e &= \frac{1}{\epsilon_r(\mathbf{r})}\nabla\left(\frac{\partial \Psi^e}{\partial z}\right) + \mu_r(\mathbf{r}) k_o^2 \Psi^e \hat{\mathbf{z}}.\end{aligned}$$

The Bessel–Fourier series is used to analytically obtain the overlap integrals over the cylindrical surface of the DR [6]. The resonant frequency obtained by using Ansoft HFSS is 1.303 GHz and that obtained by the GMMM is 1.32 GHz. As can be seen, the agreement between these two different methods is very good and the error is less than 1.3%. Moreover, since no segmentation or mesh generation is required, the GMMM is much faster than HFSS.

In the second example, we consider a two-dimensional parallel-plate resonator with a step discontinuity, as illustrated in Fig. 3. All the dimensions are in centimeters. The frequencies

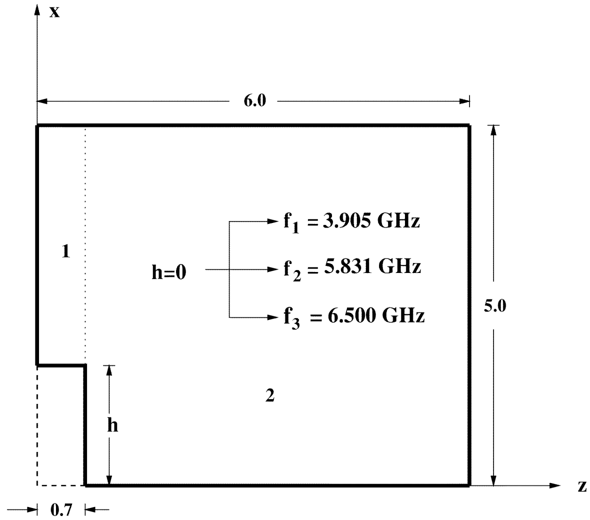

Fig. 3. Cross section of a two-dimensional parallel-plate waveguide resonator with a step.

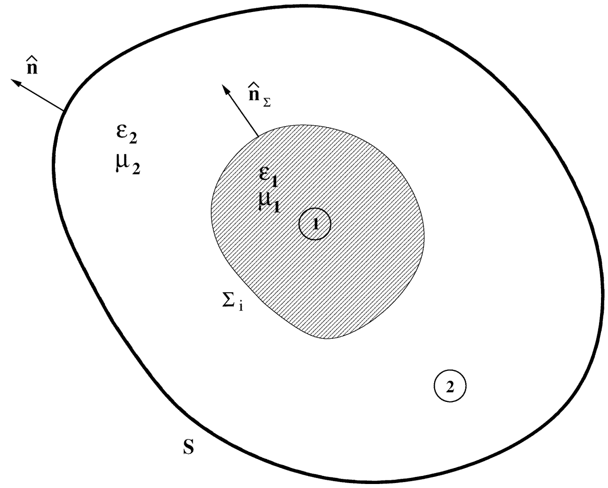

Fig. 4. Cavity resonator on which the conventional mode-matching method is applicable.

TABLE I
COMPARISON OF THE RESONANT FREQUENCIES (IN GIGAHERTZ) OF THE TWO-DIMENSIONAL RESONATOR SHOWN IN FIG. 3 OBTAINED BY HFSS AND GMMM

	HFSS			GMMM		
h	f_1	f_2	f_3	f_1	f_2	f_3
0.3	3.909	5.841	6.50	3.910	5.842	6.51
1.0	3.941	5.900	6.552	3.934	5.892	6.549
2.5	4.066	6.201	6.600	4.056	6.170	6.600
4.7	4.12	6.408	6.63	4.12	6.406	6.633

shown in this figure are the first three resonant frequencies of the TE mode (E_y, H_x, H_z) in the parallel-plate waveguide resonator without the step $(h = 0)$. By using the GMMM, we have obtained the resonant frequencies of the resonator with the step for three different values of step height h. To obtain the resonant frequencies of this structure by Ansoft HFSS, we define a three-dimensional resonator with the same cross section shown in Fig. 3 such that the dimension in the y-direction is less than the smallest dimension in the x–z–plane. The results are summarized in Table I. As illustrated, the frequencies obtained by these two different simulation methods are almost indistinguishable.

IV. CONCLUSION

In this paper, based on the variational formulation, we have developed a systematic method for obtaining resonant frequencies and field distributions inside a cavity resonator. Application of the Rayleigh–Ritz method to the variational formula results in the GMMM. In this generalized formulation, one may relax the orthogonality and some specific boundary conditions that should be met by the modes across some surfaces inside the cavity. Hence, we call it the nonorthogonal and free-boundary mode-matching method. Since the conventional mode-matching method is a special case of this generalized one, one may argue that variational principle is the mathematical basis of the former. This should not be surprising if one considers the mode-matching method as a special case of the Galerkin approach.

APPENDIX

Here, we show that, in a modal structure, if one uses the field distribution obtained by the conventional mode-matching method as a trial one in different variational formulas with $\alpha = 1/2$, the frequencies obtained by the variational formulas will be the same as that obtained by the mode-matching method. To this end, consider the cavity resonator shown in Fig. 4. The cavity walls are made of perfect electric conductors. Moreover, assume that the electromagnetic field within the cavity can be expanded in terms of the modes of regions 1 and 2 such that the modes of region 2 satisfy the boundary conditions on the cavity walls. If a trial field distribution $\mathbf{E}^a$ obtained based on the conventional mode-matching method in this cavity is used in the $\mathbf{E}$-field formulation with $\alpha = 1/2$, we have (25), shown at the bottom of this page. Using the identity $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot\nabla\times\mathbf{A} - \mathbf{A}\cdot\nabla\times\mathbf{B}$ and divergence theorem, (25) can be written as (26), shown at the top of the following page. Since $\mathbf{E}^a$ is obtained based on the mode-matching method, it satisfies homogeneous Maxwell's equations and we have

$$\mu_{1,2}^{-1}\nabla\times\mathbf{E}_{1,2}^a = -j\omega_m\mathbf{H}_{1,2}^a$$

and

$$\nabla\times\mu_{1,2}^{-1}\nabla\times\mathbf{E}_{1,2}^a = \omega_m^2\epsilon_{1,2}\mathbf{E}_{1,2}^a \tag{27}$$

$$\omega^2 = \frac{\int_{V_1+V_2}\mu_{1,2}^{-1}\left(\nabla\times\mathbf{E}_{1,2}^a\right)\cdot\left(\nabla\times\mathbf{E}_{1,2}^a\right)dv + \int_{\Sigma_i}\left(\mu_1^{-1}\nabla\times\mathbf{E}_1^a + \mu_2^{-1}\nabla\times\mathbf{E}_2^a\right)\times\left(\mathbf{E}_1^a - \mathbf{E}_2^a\right)\cdot\hat{\mathbf{n}}_\Sigma\, ds}{\int_{V_1+V_2}\epsilon_{1,2}\mathbf{E}_{1,2}^a\cdot\mathbf{E}_{1,2}^a\, dv} \tag{25}$$

$$\omega^2 = \frac{\int_{V_1+V_2} \mathbf{E}_{1,2}^a \cdot \nabla \times \mu_{1,2}^{-1} \nabla \times \mathbf{E}_{1,2}^a dv + \int_{\Sigma_i} \left(\mu_2^{-1} \nabla \times \mathbf{E}_2^a \times \mathbf{E}_1^a - \mu_1^{-1} \nabla \times \mathbf{E}_1^a \times \mathbf{E}_2^a \right) \cdot \hat{\mathbf{n}}_\Sigma \, ds}{\int_{V_1+V_2} \epsilon_{1,2} \mathbf{E}_{1,2}^a \cdot \mathbf{E}_{1,2}^a \, dv} \tag{26}$$

where ω_m is the frequency obtained by the mode-matching method. Substituting the above two equations into (26), we get

$$\omega^2 = \omega_m^2 \left[1 + j \frac{\omega_m^{-1} \int_{\Sigma_i} \left(\mathbf{E}_1^a \times \mathbf{H}_2^a - \mathbf{E}_2^a \times \mathbf{H}_1^a \right) \cdot \hat{\mathbf{n}}_\Sigma \, ds}{\int_{V_1+V_2} \epsilon_{1,2} \mathbf{E}_{1,2}^a \cdot \mathbf{E}_{1,2}^a \, dv} \right]. \tag{28}$$

If one expands $\mathbf{E}_{1,2}^a$ and $\mathbf{H}_{1,2}^a$ in terms of the modes of each region and uses the equations obtained based on the mode-matching method, one can show that the numerator in (28) vanishes. Therefore, , as stated, $\omega = \omega_m$. Note that orthogonality of the modes across Σ_i are not necessary for vanishing of the numerator in (28).

In a similar fashion, it can be shown that, if one uses the field distribution obtained by the mode-matching method as a trial one in the $\mathbf{H}$-field or mixed-field formulation with $\alpha = 1/2$, the resonant frequency will be the same as that obtained by the mode-matching method.

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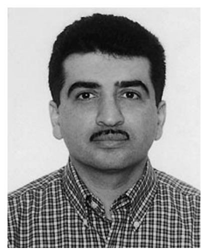
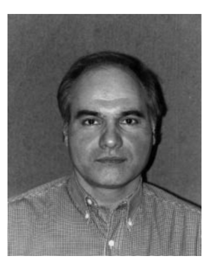


Ali M. Shams-Zadeh-Amiri (M'00) was born in Tehran, Iran, on July 13, 1961. He received the B.S. and M.S. degrees in electrical engineering from the University of Tehran, Tehran, Iran, in 1987 and 1990, respectively, and the Ph.D. degree from the University of Waterloo, Waterloo, ON, Canada, in 1997. His doctoral dissertation focused on surface-emitting distributed feedback (DFB) lasers with circular and rectangular gratings.

Until 1998, he was with Nortel Networks, Ottawa, ON, Canada, where he conducted research and development of multiple-quantum-well gain-coupled DFB lasers with second-order gratings for advanced optical communications systems. From 1999 to 2000, he was with the Department of Electrical and Computer Engineering, University of Waterloo, where he was a Research Assistant Professor involved with DR research and electromagnetic theory courses. In 2000, he joined PhotoNami Inc. Richmond Hill, ON, Canada, as a Research Scientist, and where he is currently the leader of the laser team involved with research and development of multiwavelength laser sources for optical switches in metropolitan area networks. He has three U.S. patents pending. His research interests include multiple-quantum-well DFB lasers in general and surface-emitting DFB lasers in particular, integrated optics, numerical simulation of optoelectronic devices, fiber gratings, microwave planar structures, DRs, and electromagnetic theory.



Safieddin Safavi-Naeini (S'75–M'79) received the B.Sc. degree in electrical engineering from the University of Tehran, Tehran, Iran, in 1974, and the M.Sc. and Ph.D. degrees in electrical engineering from the University of Illinois at Urbana-Champaign, in 1975 and 1979, respectively.

From 1980 to 1995, he was an Assistant and Associate Professor with the Electrical Engineering Department, University of Tehran. Since 1996, he has been an Associate Professor with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, Canada. He has been a scientific and technical consultant to a number of national and international telecommunications industrial and research organizations over the last 20 years. He has authored or coauthored over 110 papers in technical journals and conferences. His research interests and activities include numerical electromagnetics applied to analysis and design optimization of RF/microwave/millimeter-wave systems and circuits, antennas and propagation, wireless communication systems, very high-speed digital circuits, and photonics.

Sujeet K. Chaudhuri (M'79–SM'85) was born in Calcutta, India, on August 25, 1949. He received the B.E. degree (with honors) in electronics engineering from the Birla Institute of Technology and Science (BITS), Pilani, India, in 1970, the M.Tech. degree in electrical communication engineering from the Indian Institute of Technology (IIT), Delhi, India, in 1972, and the M.A.Sc. degree in microwave engineering and Ph.D. degree in electromagnetic theory from the University of Manitoba, Winnipeg, MB, Canada, in 1973 and 1977, respectively.

In 1977, he joined the University of Waterloo, Waterloo, ON, Canada, where he is currently a Professor in the Electrical and Computer Engineering Department and former Chair of the Electrical and Computer Engineering Department (1993–1998). He was also a Visiting Associate Professor with the Electrical Engineering and Computer Science Department, University of Illinois at Chicago (1981, 1984) and the National University of Singapore (1990–1991). He has been involved in contract research and consulting work with several Canadian and U.S. industries and government research organizations. His current research interests are guided-wave/electrooptic structures, planar microwave structures, DRs, optical and electromagnetic imaging, and fiber-based broad-band networks.

Dr. Chaudhuri is a member of URSI Commission B and Sigma Xi. He was the recipient of the 1998 Erskine Fellowship with the University of Canterbury, Canterbury, New Zealand.

Ramin Sabry (S'90–M'96) received the B.Sc. degree in electronics and communication engineering from Tehran University, Tehran, Iran, in 1991, and the M.A.Sc. (microwaves and electromagnetics) and Ph.D. degrees (high-energy electrodynamics and photonics/electrooptics) in electrical engineering from the University of Waterloo, Waterloo, ON, Canada, in 1991 and 1996, respectively.

Since 1996, he has been with Ericsson Canada Inc., Mississauga, ON, Canada, where he conducts research on various aspects of wireless communications, e.g., RF components, systems, and propagation. He has been Ericsson Canada Inc.'s Technical Advisor/Director for radio research at the Center for Wireless Communication (CWC), University of Waterloo, and holds an adjunct faculty position with the Electrical and Computer Engineering Department, University of Waterloo. His current research interests are in radio/RF for wireless communications, electromagnetics, and photonics.